Mathematik LK 12 M2, HÜ Nr. 2 – Integrationsmethoden – Lösung N **01.12.2016**

Aufgabe 1: Berechne die folgenden Integrale mit einer Integrationsmethode deiner Wahl.

1.1 $\int \cos^2(x)\,dx = \int \cos(x)\cdot\cos(x)\,dx = \cos(x)\sin(x) - \int -\sin(x)\sin(x)\,dx$

$\Leftrightarrow \int \cos^2(x)\,dx = \cos(x)\sin(x) + C_1 + \int \sin^2(x)\,dx = \cos(x)\sin(x) + \int 1 - \cos^2(x)\,dx$

$\Leftrightarrow \int \cos^2(x)\,dx = \cos(x)\sin(x) + C_1 + \int 1\,dx - \int \cos^2(x)\,dx \quad | + \int \cos^2(x)\,dx$

$\Leftrightarrow 2\int \cos^2(x)\,dx = \cos(x)\sin(x) + C_1 + x + C_2 \quad | \ :2$

$\Leftrightarrow \int \cos^2(x)\,dx = \mathbf{\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x + C} = \frac{1}{2}\sin(2x) + \frac{1}{2}x + C$

1.2 $\int \frac{2x-1}{x^3+3x^2-4}\,dx$

Suche Nullstellen des Nenners: $x_1 = 1$ durch Probieren.

Polynomdivision:	Finde weitere NST:
$(x^3+3x^2 \quad -4):(x-1) = x^2+4x+4$ $-(x^3 - x^2)$ $4x^2+0x$ $-(4x^2-4x)$ $4x-4$ $-(4x-4)$ 0	$0 = x_{2/3}^2 + 4x_{2/3} + 4 = (x_{2/3}+2)^2 \Rightarrow x_{2/3} = -2$ $x_2 = -2$ ist doppelte NST.

Also

$$\frac{2x-1}{x^3+3x^2-4} = \frac{2x-1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{A(x+2)^2}{(x-1)(x+2)^2} + \frac{B(x-1)(x+2)}{(x-1)(x+2)^2} + \frac{C(x-1)}{(x-1)(x+2)^2}$$

Vergleich der Zähler:

$$2x-1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

Für $x=1$ folgt: $2\cdot1-1 = A(1+2)^2 + B(1-1)(1+2) + C(1-1) \Leftrightarrow 1 = A\cdot3^2+0+0 \Leftrightarrow A = \frac{1}{9}$

Für $x=-2$ folgt:

$2\cdot(-2)-1 = A(2-2)^2 + B(-2-1)(-2+2) + C(-2-1) \Leftrightarrow -5 = 0+0+C\cdot(-3) \Leftrightarrow C = \frac{5}{3}$

Setze $x=-1$ ein:

$$2\cdot(-1)-1 = A((-1)+2)^2 + B(-1-1)(-1+2) + C(-1-1) = -3 = \frac{1}{9}\cdot1^2 + B\cdot(-2) + \frac{5}{3}\cdot(-2)$$

$$\Leftrightarrow -3 = \frac{1}{9} - 2B - \frac{10}{3} \Leftrightarrow -\frac{27}{9} = \frac{1}{9} - 2B - \frac{30}{9} \Leftrightarrow \frac{2}{9} = -2B \Leftrightarrow -\frac{1}{9} = B$$

Also

$$\int \frac{2x-1}{x^3+3x^2-4}\,dx=\int \frac{1}{9(x-1)}-\frac{1}{9(x+2)}+\frac{5}{3(x+2)^2}\,dx=\int \frac{1}{9(x-1)}\,dx-\int \frac{1}{9(x+2)}\,dx+\int \frac{5}{3(x+2)^2}\,dx$$
$$=\mathbf{\frac{\ln(x-1)-\ln(x+2)}{9}-\frac{5}{3\cdot(x+2)}+C}$$

1.3 $\int \frac{1}{x^2\cdot\sqrt{1-x^2}}\,dx$ (Substituiere $x=\frac{1}{z}$)

$$\frac{dx}{dz}=-\frac{1}{z^2}\Leftrightarrow dx=-\frac{1}{z^2}\,dz$$

$$\int \frac{1}{x^2\cdot\sqrt{1-x^2}}\,dx=\int \frac{1}{\left(\frac{1}{z}\right)^2\cdot\sqrt{1-\left(\frac{1}{z}\right)^2}}\cdot\left(-\frac{1}{z^2}\right)dz=-\int \frac{1}{\sqrt{1-\frac{1}{z^2}}}\,dz=-\int \frac{1}{\sqrt{\frac{z^2-1}{z^2}}}\,dz=-\int \sqrt{\frac{z^2}{z^2-1}}\,dz$$

$=-\int \frac{z}{\sqrt{z^2-1}}\,dz$ Substitution: $u=z^2-1$ $\quad \frac{du}{dz}=2z\Leftrightarrow dz=\frac{1}{2}z\,du$

$-\int \frac{z}{\sqrt{z^2-1}}\,dz=-\int \frac{z}{2z\sqrt{u}}\,du=-\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du=-\frac{1}{2}\cdot 2\sqrt{u}=-\sqrt{u}$ Rücksubstitution: $u=z^2-1$

$\Rightarrow -\int \frac{z}{\sqrt{z^2-1}}\,dz=-\sqrt{z^2-1}$ Rücksubstitution: $z=\frac{1}{x}\Rightarrow$ $\quad \int \frac{1}{x^2\cdot\sqrt{1-x^2}}\,dx=-\sqrt{\frac{1}{x^2}-1}$