Mathematik LK 12 M2, HÜ Nr. 2 – Integrationsmethoden – Lösung **28.11.2016**

<u>Aufgabe 1:</u> Berechne die folgenden Integrale mit einer Integrationsmethode deiner Wahl.

<u>1.1</u> $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

Substitution: $x = \sin(z)$ $\quad \frac{dx}{dz} = \cos(z) \Leftrightarrow dx = \cos(z)\,dz$

Grenzen: $\sin(\bar{a}) = 0 \Rightarrow \bar{a} = 0 \quad \sin(\bar{b}) = \frac{1}{2} \Rightarrow \bar{b} = \frac{\pi}{6}$

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2(z)}} \cdot \cos(z)\,dz = \int_0^{\frac{\pi}{6}} \frac{1}{\cos(z)} \cdot \cos(z)\,dz = \int_0^{\frac{\pi}{6}} 1\,dz = [z]_0^{\pi/6} = \frac{\pi}{6}$$

<u>1.2</u> $\int \frac{1}{1+e^x}\,dx$ Substitution: $z = e^x \;\Leftrightarrow\; x = \ln(z) \;\Rightarrow\; \frac{dz}{dx} = e^x \;\Leftrightarrow\; dx = \frac{dz}{e^x}$

$$\Rightarrow \int \frac{1}{1+e^x}\,dx = \int \frac{1}{1+z}\frac{dz}{e^x} = \int \frac{1}{1+z}\frac{dz}{e^{\ln(z)}} = \int \frac{1}{z(1+z)}\,dz$$

Partialbruchzerlegung: $\frac{1}{z(1+z)} = \frac{A}{z} + \frac{B}{1+z} = \frac{A(1+z)}{z(1+z)} + \frac{Bz}{z(1+z)}$

Vergleiche Zähler: $1 = A(1+z) + Bz$

Setze $z = -1 \;\Rightarrow\; 1 = A(1+(-1)) + B\cdot(-1) \;\Leftrightarrow\; 1 = -B \;\Leftrightarrow\; B = -1$

Setze $z = 0 \;\Rightarrow\; 1 = A(1+0) + B\cdot 0 \;\Leftrightarrow\; 1 = A$

Also $\int \frac{1}{z(1+z)}\,dz = \int \frac{1}{z} - \frac{1}{1+z}\,dz = \int \frac{1}{z}\,dz - \int \frac{1}{1+z}\,dz = \ln(z) + C_1 - \ln(1+z) - C_2$ mit $C = C_1 + C_2$

$$= \ln(z) - \ln(1+z) + C = \ln(e^x) - \ln(1+e^x) + C = x - \ln(1+e^x) + C \quad \left(= \ln\left(\frac{e^x}{1+e^x}\right) + C \right)$$

<u>1.3</u> $\int a^x e^x \, dx$ (Tipp: Ersetze *x* durch einen Ausdruck, der den Term e^x vereinfacht).

Substitution: $x = \ln(z)$ Damit $\frac{dx}{dz} = \frac{1}{z} \Leftrightarrow dx = \frac{dz}{z}$

$$\int a^x e^x \, dx = \int \frac{a^{\ln(z)} \cdot e^{\ln(z)}}{z} dz = \int \frac{\left(e^{\ln(a)}\right)^{\ln(z)} \cdot z}{z} dz = \int e^{\ln(z) \cdot \ln(a)} dz = \int \left(e^{\ln(z)}\right)^{\ln(a)} dz$$

$$= \int z^{\ln(a)} dz = \frac{1}{\ln(a)+1} \cdot z^{\ln(a)+1} + C$$

Rücksubstitution: $z = e^x$

$$\frac{z^{\ln(a)+1}}{\ln(a)+1} = \frac{e^{x^{\ln(a)+1}}}{\ln(a)+1} = \frac{e^{x \cdot \ln(a)+x}}{\ln(a)+1} = \frac{e^{x \cdot \ln(a)} \cdot e^x}{\ln(a)+1} = \frac{a^x \cdot e^x}{\ln(a)+1}$$

Also ist

$$\int a^x e^x \, dx = \frac{a^x \cdot e^x}{\ln(a)+1} + C$$