

Aufgabe 1: Bestimme die Grenzwerte der folgenden Folgen mit Hilfe der Grenzwertsätze:

1.1 $a_n = -2n^3 + \frac{n^5}{10} + 2n^2 + 100$

$$\lim_{n \rightarrow \infty} \left(-2n^3 + \frac{n^5}{10} + 2n^2 + 100 \right) = \lim_{n \rightarrow \infty} \left(n^5 \left(\frac{-2}{n^2} + \frac{1}{10} + \frac{2}{n^3} + \frac{100}{n^5} \right) \right) = \left(\lim_{n \rightarrow \infty} n^5 \right) \left(-0 + \frac{1}{10} + 0 + 0 \right) = +\infty$$

1.2 $a_n = \frac{3(n^2-1)}{4n^2+1}$

$$\lim_{n \rightarrow \infty} \frac{3(n^2-1)}{4n^2+1} = \lim_{n \rightarrow \infty} \frac{3n^2 \left(1 - \frac{1}{n^2} \right)}{4n^2 \left(1 + \frac{1}{4n^2} \right)} = \frac{3}{4} \cdot \lim_{n \rightarrow \infty} \frac{\left(1 - \frac{1}{n^2} \right)}{\left(1 + \frac{1}{4n^2} \right)} = \frac{3}{4} \frac{1-0}{1+0} = \frac{3}{4}$$

1.3 $a_n = \frac{\frac{2}{n^3} + \frac{2}{n^2} + \frac{2}{n}}{\frac{1}{n} + \frac{1}{n^3} + \frac{1}{n^2}} = \frac{2 \left(\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} \right)}{\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3}} = 2$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} 2 = 2$$