

Aufgabe 1: Bestimme die Grenzwerte der folgenden Folgen mit Hilfe der Grenzwertsätze:

1.1 $a_n = -n^2 + \frac{1}{10}n^3 + 2n + 10$

$$\lim_{n \rightarrow \infty} \left(-n^2 + \frac{1}{10}n^3 + 2n + 10 \right) = \lim_{n \rightarrow \infty} \left(n^3 \left(-\frac{1}{n} + \frac{1}{10} + \frac{2}{n^2} + \frac{10}{n^3} \right) \right) = \left(\lim_{n \rightarrow \infty} n^3 \right) \left(-0 + \frac{1}{10} + 0 + 0 \right) = +\infty$$

1.2 $a_n = \frac{2n^2 - 1}{3(n^2 + 1)}$

$$\lim_{n \rightarrow \infty} \frac{2n^2 - 1}{3(n^2 + 1)} = \lim_{n \rightarrow \infty} \frac{2n^2 \left(1 - \frac{1}{2n^2} \right)}{3n^2 \left(1 + \frac{1}{n^2} \right)} = \frac{2}{3} \cdot \lim_{n \rightarrow \infty} \frac{\left(1 - \frac{1}{2n^2} \right)}{\left(1 + \frac{1}{n^2} \right)} = \frac{2}{3} \frac{1 - 0}{1 + 0} = \frac{2}{3}$$

1.3 $a_n = \frac{\frac{1}{n^2} + \frac{1}{n} + \frac{1}{n^3}}{\frac{2}{n^3} + \frac{2}{n} + \frac{2}{n^2}} = \frac{\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3}}{2 \left(\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} \right)} = \frac{1}{2}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{2} = \frac{1}{2}$$