Mathematik LK 11 M1, AB 17 Ableitungsregeln 03 – Lösung 06.06.2016

1.5 $f(x)=\frac{x^4-2x^3-2x^2}{(x-1)^2}$ Berechne die ersten drei Ableitungen.

$\left(Kontrolllösung: f'(x)=\frac{2x^4-6x^3+6x^2+4x}{(x-1)^3}\right)$ *(Benutze ggf. die Kontrollösung für f''(x).)*

$\left(Kontrolllösung: f''(x)=\frac{2x^4-8x^3+12x^2-20x-4}{(x-1)^4}\right)$

$$f(x)=\frac{x^4-2x^3-2x^2}{(x-1)^2}=\frac{u(x)}{v(x)} \Rightarrow u'(x)=4x^3-6x^2-4x \; ; \; v'(x)=1\cdot 2(x-1)=2(x-1)$$

$$f'(x)=\frac{(4x^3-6x^2-4x)(x-1)^2-(x^4-2x^3-2x^2)\cdot 2(x-1)}{(x-1)^4}=\frac{(4x^3-6x^2-4x)(x-1)-2(x^4-2x^3-2x^2)}{(x-1)^3}$$

$$=\frac{4x^4-6x^3-4x^2-4x^3+6x^2+4x-2x^4+4x^3+4x^2}{(x-1)^3}=\frac{2x^4-6x^3+6x^2+4x}{(x-1)^3}=\frac{u_1(x)}{v_1(x)}$$

$$u_1'(x)=8x^3-18x^2+12x+4 \; ; \; v_1'(x)=3(x-1)^2$$

$$f''(x)=\frac{(8x^3-18x^2+12x+4)\cdot(x-1)^3-(2x^4-6x^3+6x^2+4x)\cdot 3(x-1)^2}{(x-1)^6}$$

$$=\frac{(8x^3-18x^2+12x+4)\cdot(x-1)-(2x^4-6x^3+6x^2+4x)\cdot 3}{(x-1)^4}$$

$$=\frac{8x^4-18x^3+12x^2+4x-8x^3+18x^2-12x-4-6x^4+18x^3-18x^2-12x}{(x-1)^4}$$

$$=\frac{2x^4-8x^3+12x^2-20x-4}{(x-1)^4}=\frac{u_2(x)}{v_2(x)}$$

$$u_2'(x)=8x^3-24x^2+24x-20 \quad v_2'(x)=4(x-1)^3$$

$$f'''(x)=\frac{(8x^3-24x^2+24x-20)(x-1)^4-(2x^4-8x^3+12x^2-20x-4)\cdot 4(x-1)^3}{(x-1)^8}$$

$$=\frac{(8x^3-24x^2+24x-20)(x-1)-(2x^4-8x^3+12x^2-20x-4)\cdot 4}{(x-1)^5}$$

$$=\frac{8x^4-24x^3+24x^2-20x-8x^3+24x^2-24x+20-8x^4+32x^3-48x^2+80x+16}{(x-1)^5}$$

$$=\frac{36x+36}{(x-1)^5}=\frac{36(x+1)}{(x-1)^5}$$