Mathematatik LK 11 M2, AB 12 – Nullstellenberechnung **14.01.2016**

Aufgabe 1: Berechne die Nullstellen der folgenden Funktionen:

1.1 $f(x)=-\frac{1}{2}x-8 \quad 0=-\frac{1}{2}x_n-8 \quad | +8 \quad \Leftrightarrow \quad 8=-\frac{1}{2}x_n \quad | \cdot(-2) \quad \Leftrightarrow \quad \mathbf{-16=x_n}$

1.2 $f(x)=-\frac{1}{2}x^2-3x+8 \quad 0=-\frac{1}{2}x_n^2-3x_n+8 \quad | \cdot(-2) \quad \Leftrightarrow \quad 0=x_n^2+6x_n-16$

p-q-Formel: $x_{1/2}=-3\pm\sqrt{3^2+16}=3\pm\sqrt{25}=-3\pm5 \Rightarrow \mathbf{x_1=-8 \;;\; x_2=2}$

1.3 $f(x)=-\frac{1}{2}x^4-2x^2+16 \quad 0=-\frac{1}{2}x_n^4-2x_n^2+16$ Substituiere $z=x^2$

$\Rightarrow 0=-\frac{1}{2}z_n^2-2z_n+16 \quad | \cdot(-2) \quad \Leftrightarrow \quad 0=z_n^2+4z_n-32$

p-q-Formel: $z_{1/2}=-2\pm\sqrt{2^2+32}=-2\pm\sqrt{36}=-2\pm6 \Rightarrow z_1=-8 \;;\; z_2=4$ Rücksubstitution $x=\pm\sqrt{z}$

$x_{1/2}=\pm\sqrt{z_2}=\pm\sqrt{4} \Leftrightarrow \mathbf{x_1=-2; x_2=2}$
$x_{3/4}=\pm\sqrt{z_1}=\pm\sqrt{-8}$ geht nicht, also keine weiteren Nullstellen.

1.4 $f(x)=-\frac{1}{2}x^4+2x^3+6x^2 \quad 0=-\frac{1}{2}x_n^4+2x_n^3+6x_n^2=-\frac{1}{2}x_n^2\cdot\left(x_n^2-4x_n-12\right) \quad \Rightarrow \mathbf{x_1}=0$

Betrachte Klammer: $0=x_n^2-4x_n-12$
p-q-Formel: $x_{2/3}=2\pm\sqrt{(-2)^2+12}=2\pm\sqrt{16}=2\pm4 \Rightarrow \mathbf{x_2=-2 \;;\; x_3=6}$

1.5 $f(x)=2x^3-4x^2-22x+24 \quad 0=2x_n^3-4x_n^2-22x_n+24 \quad \mathbf{x_1=1}$ durch Probieren

$$
\begin{array}{l}
(2x^3-4x^2-22x+24):(x-1)=2x^2-2x-24 \\
\underline{-(2x^3-2x^2)} \\
\quad -2x^2-22x+24 \\
\quad \underline{-(-2x^2+\ 2x)} \\
\qquad -24x+24 \\
\qquad \underline{-(-24x+24)} \\
\qquad\qquad 0
\end{array}
$$

Also $0=(x_n-1)(2x_n^2-2x_n-24)$ Betrachte 2. Klammer:

$0=2x_n^2-2x_n-24 \quad | :2 \quad \Leftrightarrow \quad 0=x_n^2-x_n-12$

p-q-Formel: $x_{2/3}=\frac{1}{2}\pm\sqrt{\left(-\frac{1}{2}\right)^2+12}=\frac{1}{2}\pm\sqrt{12{,}25}=0{,}5\pm3{,}5 \Rightarrow \mathbf{x_2=-3 \;;\; x_3=4}$

1.6 $f(x)=x^4+3x^3-3x^2-7x+6$ $\quad 0=x_n^4+3x_n^3-3x_n^2-7x_n+6$ $\quad \mathbf{x_1=1}$ durch Probieren

$$
\begin{aligned}
&(x^4+3x^3-3x^2-7x+6):(x-1)=x^3+4x^2+x-6\\
&\underline{-(x^4-x^3)}\\
&\quad 4x^3-3x^2-7x+6\\
&\quad \underline{-(4x^3-4x^2)}\\
&\qquad x^2-7x+6\\
&\qquad \underline{-(x^2-\ \ x)}\\
&\qquad\quad -6x+6\\
&\qquad\quad \underline{-(-6x+6)}\\
&\qquad\qquad 0 \quad \text{Also} \quad 0=(x_n-1)(x_n^3+4x_n^2+x_n-6)
\end{aligned}
$$

Betrachte 2. Klammer: $0=x_n^3+4x_n^2+x_n-6$ $\quad \mathbf{x_1=1}$ durch Probieren (x_1 ist also doppelte NST)

$$
\begin{aligned}
&(x^3+4x^2+\ \ x-6):(x-1)=x^2+5x+6\\
&\underline{-(x^3-\ \ x^2)}\\
&\quad 5x^2+\ \ x-6\\
&\quad \underline{-(5x^2-5x)}\\
&\qquad 6x-6\\
&\qquad \underline{-(6x-6)}\\
&\qquad\quad 0 \quad \text{Also} \quad 0=(x_n-1)^2(x_n^2+5x_n+6)
\end{aligned}
$$

Betrachte 2. Klammer: $0=x_n^2+5x_n+6$

p-q-Formel:

$$x_{2/3}=-2{,}5\pm\sqrt{(2{,}5)^2-6}=-2{,}5\pm\sqrt{6{,}25-6}=-2{,}5\pm\sqrt{0{,}25}=-2{,}5\pm0{,}5 \;\Rightarrow\; \mathbf{x_2=-3\ ;\ x_3=-2}$$

1.7 $f(x)=2x^4+2x^3-7{,}5x^2-9x$

$$0=2x_n^4+2x_n^3-7{,}5x_n^2-9x_n \;\Leftrightarrow\; 0=2x_n\cdot\left(x_n^3+x_n^2-3{,}75x_n-4{,}5\right) \;\Rightarrow\; \mathbf{x_1=0}$$

Betrachte Klammer: $0=x_n^3+x_n^2-3{,}75x_n-4{,}5$ $\quad ;\mathbf{x_2=2}$ durch Probieren

$$
\begin{aligned}
&\left(x^3+\ \ x^2-\frac{15}{4}x-\frac{9}{2}\right):(x-2)=x^2+3x+\frac{9}{4}\\
&-\underline{(x^3-2x^2)}\\
&\quad 3x^2-\frac{15}{4}x-\frac{9}{2}\\
&\quad -\underline{(3x^2-\ 6x)}\\
&\qquad \frac{9}{4}x-\frac{9}{2}\\
&\qquad \underline{-\left(\frac{9}{4}x-\frac{9}{2}\right)}\\
&\qquad\quad 0
\end{aligned}
$$

Also $0=2x_n\cdot(x_n-2)\left(x_n^2+3x_n+\frac{9}{4}\right)$

Betrachte 2. Klammer: $0=x_n^2+3x_n+\frac{9}{4}=(x+1{,}5)^2 \Rightarrow \mathbf{x_3=-1{,}5}$ doppelte NST. Oder p-q-Formel:

$\mathbf{x_{3/4}}=-1\pm\sqrt{(1{,}5)^2-2{,}25}=-1{,}5\pm\sqrt{2{,}25-2{,}25}=-1{,}5\pm\sqrt{0}=\mathbf{-1{,}5}$

1.8 $f(x)=x^4+x^3-2x^2-16x+16$ $\quad 0=x_n^4+x_n^3-2x_n^2-16x_n+16$ $\quad \mathbf{x_1=1}$ durch Probieren

$$
\begin{array}{l}
(x^4+x^3-2x^2-16x+16):(x-1)=x^3+2x^2-16 \\
-\underline{(x^4-x^3)} \\
\quad 2x^3-2x^2-16x+16 \\
\quad -\underline{(2x^3-2x^2)} \\
\qquad\quad -16x+16 \\
\qquad\quad -\underline{(-16x+16)} \\
\qquad\qquad\qquad 0
\end{array}
$$

Also $0=(x_n-1)(x_n^3+2x_n^2-16)$ Betrachte 2. Klammer:

$0=x_n^3+2x_n^2-16$ $\quad \mathbf{x_2=2}$ durch Probieren

$$
\begin{array}{l}
(x^3+2x^2+0x-16):(x-2)=x^2+4x+8 \\
-\underline{(x^3-2x^2)} \\
\quad 4x^2+0x\ -16 \\
\quad -\underline{(4x^2-8x)} \\
\qquad\quad 8x-16 \\
\qquad\quad -\underline{(8x-16)} \\
\qquad\qquad\quad 0
\end{array}
$$

Also $0=(x_n-1)(x_n-2)(x_n^2+4x_n+8)$ Betrachte 3. Klammer:

$0=x_n^2+4x_n+8$ p-q-Formel:

$\mathbf{x_{3/4}}=-2\pm\sqrt{2^2-8}=-2\pm\sqrt{-4}$ Also keine weiteren Nullstellen.

1.9 $f(x)=0{,}5x^7+x^6-5{,}5x^5-6x^4+18x^3$

$0=0{,}5x_n^7+x_n^6-5{,}5x_n^5-6x_n^4+18x_n^3 \Leftrightarrow 0=0{,}5x_n^3\cdot\left(x_n^4+2x_n^3-11x_n^2-12x_n+36\right) \Rightarrow \mathbf{x_1=0}$

Betrachte Klammer: $0=x_n^4+2x_n^3-11x_n^2-12x_n+36$ $\quad \mathbf{x_2=2}$ durch Probieren

$$
\begin{array}{l}
(x^4+2x^3-11x^2-12x+36):(x-2)=x^3+4x^2-3x-18 \\
-\underline{(x^4-2x^3)} \\
\quad 4x^3-11x^2-12x+36 \\
\quad -\underline{(4x^3-\ 8x^2)} \\
\qquad\quad 3x^2-12x+36 \\
\qquad\quad -\underline{(-3x^2+\ 6x)} \\
\qquad\qquad\quad -18x+36 \\
\qquad\qquad\quad -\underline{(-18x+36)} \\
\qquad\qquad\qquad\quad 0
\end{array}
$$

Also $0=(x_n-2)(x_n^3+4x_n^2-3x_n-18)$ Betrachte 2. Klammer:

$0=x_n^3+4x_n^2-3x_n-18$ $\mathbf{x_2=2}$ durch Probieren (x_2 ist also doppelte NST)

$$
\begin{aligned}
&(x^3+4x^2-3x-18):(x-2)=x^2+6x+9\\
&-\underline{(x^3-2x^2)}\\
&\quad 6x^2-3x-18\\
&\quad -\underline{(6x^2-12x)}\\
&\qquad 9x-18\\
&\qquad -\underline{(9x-18)}\\
&\qquad\quad 0
\end{aligned}
$$

Also $0=(x_n-2)^2(x_n^2+6x_n+9)$ Betrachte 2. Klammer:

$0=x_n^2+6x_n+9 \quad\Leftrightarrow\quad 0=(x_n+3)^2 \quad\Rightarrow\quad \mathbf{x_3=-3}$ Doppelte NST

1.10 $f(x)=x^5+11x^4-2x^3-148x^2-296x-160$

$0=x_n^5+11x_n^4-2x_n^3-148x_n^2-296x_n-160$ $\mathbf{x_1=-1}$ durch Probieren

$$
\begin{aligned}
&(x^5+11x^4-2x^3-148x^2-296x-160):(x+1)=x^4+10x^3-12x^2-136x-160\\
&-\underline{(x^5+x^4)}\\
&\quad 10x^4-2x^3-148x^2-296x-160\\
&\quad -\underline{(10x^4+10x^3)}\\
&\qquad -12x^3-148x^2-296x-160\\
&\qquad -\underline{(-12x^3-12x^2)}\\
&\qquad\quad -136x^2-296x-160\\
&\qquad\quad -\underline{(-136x^2-136x)}\\
&\qquad\qquad -160x-160\\
&\qquad\qquad -\underline{(-160x-160)}\\
&\qquad\qquad\quad 0
\end{aligned}
$$

Also $0=(x_n+1)(x_n^4+10x_n^3-12x_n^2-136x_n-160)$ Betrachte 2. Klammer:

$0=x_n^4+10x_n^3-12x_n^2-136x_n-160$ $\mathbf{x_2=-2}$ durch Probieren

$$
\begin{aligned}
&(x^4+10x^3-12x^2-136x-160):(x+2)=x^3+8x^2-28x-80\\
&-\underline{(x^4+2x^3)}\\
&\quad 8x^3-12x^2-136x-160\\
&\quad -\underline{(8x^3+16x^2)}\\
&\qquad -28x^2-136x-160\\
&\qquad -\underline{(-28x^2-56x)}\\
&\qquad\quad -80x-160\\
&\qquad\quad -\underline{(-80x-160)}\\
&\qquad\qquad 0
\end{aligned}
$$

Also $0=(x_n+1)(x_n+2)(x_n^3+8x_n^2-28x_n-80)$ Betrachte 3. Klammer:

$0=x_n^3+8x_n^2-28x_n-80$ $\mathbf{x_2=-2}$ durch Probieren, also doppelte NST

$$
\begin{aligned}
&(x^3+8x^2-28x-80):(x+2)=x^2+6x-40\\
&-\underline{(x^3+2x^2)}\\
&\quad 6x^2-28x-80\\
&\quad -\underline{(6x^2+12x)}\\
&\qquad -40x-80\\
&\qquad -\underline{(-40x-80)}\\
&\qquad\qquad 0
\end{aligned}
$$

Also $0=(x_n+1)(x_n+2)^2(x_n^2+6x_n-40)$ Betrachte 3. Klammer:

$0=x_n^2+6x_n-40$ p-q-Formel:

$x_{2/3}=-3\pm\sqrt{(3)^2+40}=-3\pm\sqrt{49}=-3\pm 7 \Rightarrow \mathbf{x_2=-10}$ **;** $\mathbf{x_3=4}$