Mathematik LK 11 M1, AB 03 – Grenzwertsätze – Lösung **18.09.2013**

<u>Aufgabe 1:</u> Bestimme die Grenzwerte der folgenden Folgen mit Hilfe der Grenzwertsätze:

a) $a_n = -n^3 + 20n^2 + 2n + 10$

$$\lim_{n\to\infty}\left(-n^3+20n^2+2n+10\right)=\lim_{n\to\infty}\left(-n^3\cdot\left(1+\frac{20}{n}-\frac{2}{n^2}-\frac{10}{n^3}\right)\right)=\left(\lim_{n\to\infty}(-1)\cdot n^3\right)\cdot(1+0-0-0)$$
$$=-1\cdot\left(\lim_{n\to\infty}n^3\right)\cdot 1=-\infty$$

b) $a_n = \frac{1}{n^3} + 20n^2 + 2n + 10$

$$\lim_{n\to\infty}\left(\frac{1}{n^3}+20n^2+2n+10\right)=\lim_{n\to\infty}\left(n^2\cdot\left(\frac{1}{n^5}+20+\frac{2}{n}+\frac{10}{n^2}\right)\right)=\left(\lim_{n\to\infty}n^2\right)\cdot(0+20+0+0)=\infty$$

c) $a_n = \frac{1}{1000}n^{12} + 1000n^3 - 1000n^2 + n - 2$

$$\lim_{n\to\infty}\left(\frac{1}{1000}n^{12}+1000n^3-1000n^2+n-2\right)=\lim_{n\to\infty}\left(n^{12}\cdot\left(\frac{1}{1000}+\frac{1000}{n^9}+\frac{1000}{n^{10}}+\frac{1}{n^{11}}-\frac{2}{n^{12}}\right)\right)$$
$$=\left(\lim_{n\to\infty}n^{12}\right)\cdot\left(\frac{1}{1000}+0+0+0+0\right)=\infty$$

d) $a_n = \frac{n^2+6n}{n^2+2n}$

$$\lim_{n\to\infty}\left(\frac{n^2+6n}{n^2+2n}\right)=\lim_{n\to\infty}\left(\frac{n^2\left(1+\frac{6}{n}\right)}{n^2\cdot\left(1+\frac{2}{n}\right)}\right)=\lim_{n\to\infty}\left(\frac{1+\frac{6}{n}}{1+\frac{2}{n}}\right)=\frac{\lim_{n\to\infty}\left(1+\frac{6}{n}\right)}{\lim_{n\to\infty}\left(1+\frac{2}{n}\right)}=\frac{1}{1}=1$$

e) $a_n = \frac{n^2+6n+9}{n-3}$

$$\lim_{n\to\infty}\left(\frac{n^2+6n+9}{n-3}\right)=\lim_{n\to\infty}\left(\frac{n\cdot\left(n+6+\frac{9}{n}\right)}{n\cdot\left(1-\frac{3}{n}\right)}\right)=\frac{\lim_{n\to\infty}\left(n+6+\frac{9}{n}\right)}{\lim_{n\to\infty}\left(1-\frac{3}{n}\right)}=\frac{\lim_{n\to\infty}\left(n+6+\frac{9}{n}\right)}{1}=\infty$$

f) $a_n = \frac{2n^{12}-6n^3+9}{n^{12}-6n^{10}}$

$$\lim_{n\to\infty}\left(\frac{2n^{12}-6n^3+9}{n^{12}-6n^{10}}\right)=\lim_{n\to\infty}\left(\frac{n^{12}\cdot\left(2-\frac{6}{n^9}+\frac{9}{n^{12}}\right)}{n^{12}\cdot\left(1-\frac{6}{n^2}\right)}\right)=\frac{2-0+0}{1-0}=2$$

g) $a_n = \frac{\frac{1}{n^3}}{\frac{1}{n^2}}$

$$\lim_{n\to\infty}\left(\frac{\frac{1}{n^3}}{\frac{1}{n^2}}\right) = \lim_{n\to\infty}\left(\frac{n^2}{n^3}\right) = \frac{\lim_{n\to\infty} 1}{n} = 0$$

h) $a_n = \frac{(n+1)(n^2+6\mathrm{n}+9)}{n(n^2-1)}$

$$\lim_{n\to\infty}\left(\frac{(n+1)(n^2+6\mathrm{n}+9)}{n(n^2-1)}\right) = \lim_{n\to\infty}\left(\frac{(n+1)(n^2+6\mathrm{n}+9)}{n(n+1)(n-1)}\right) = \lim_{n\to\infty}\left(\frac{n^2+6\mathrm{n}+9}{n^2-n}\right) = \lim_{n\to\infty}\left(\frac{n^2\cdot\left(1+\frac{6}{n}+\frac{9}{n^2}\right)}{n^2\cdot\left(1-\frac{1}{n}\right)}\right)$$

$$= \frac{1+0+0}{1-0} = 1$$