Mathematik LK 11 M1, AB 13 – Exponentialgleichungen – Lösung **11.04.2013**

Aufgabe 1: Löse die folgenden Exponentialgleichungen.

a) $16^x - 512 \cdot 2^{2x} + 65536 = 0$

NR: $512 \cdot 2^{2x} = 512 \cdot \left(2^2\right)^x = 512 \cdot 4^x \qquad 16^x = \left(4^2\right)^x = 4^{2x} = 4^{x \cdot 2} = \left(4^x\right)^2$

$\left(4^x\right)^2 - 512 \cdot 4^x + 65536 = 0$ Substitution $u = 4^x$

$\Rightarrow u^2 - 512\,u + 65536 = 0$ p-q-Formel:

$u_{1/2} = 256 \pm \sqrt{256^2 - 65536} = 256 \pm \sqrt{(2^8)^2 - 2^{16}} = 256 \pm 0$

Rücksubstitution: $u = 4^x \Leftrightarrow x = \log_4(u) = \log_4(256) = \mathbf{4}$

b) $5^{6x} - 50 \cdot 125^x + 625 = 0$

NR: $5^{6x} = 5^{3 \cdot 2x} = \left(5^3\right)^{2x} = 125^{2x} = \left(125^x\right)^2$

$\left(125^x\right)^2 - 50 \cdot 125^x + 625 = 0$ Substitution $u = 125^x$

$\Rightarrow u^2 - 50\,u + 625 = 0$ p-q-Formel:

$u_{1/2} = 25 \pm \sqrt{25^2 - 625} = 25 \pm \sqrt{0} = 25 \pm 0$

Rücksubstitution: $u = 125^x \Leftrightarrow \boldsymbol{x} = \log_{125}(u) = \log_{125}(25) = \dfrac{\ln(25)}{\ln(125)} = \dfrac{\mathbf{2}}{\mathbf{3}}$